STSCI 3740/5740 Machine Learning and Data Mining $_{\rm Spring\ 2024}$

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Homework 3, due November 28, 11:59pm

Problem 1 (10 points)

In the lectures, we used logistic regression to predict the probability of **default** using **income** and **balance** on the **Default** data set, which is contained in the package **ISLR**. You may use

library(ISLR) data=Default

In this problem, you will estimate the corresponding test error based on the validation set approach and cross-validation.

- 1. Fit a logistic regression model that uses **income** and **balance** to predict **default**.
- 2. Use a validation set approach to estimate the test error in this model. To do so, consider the following steps
 - (a) Use set.seed(1) to make results reproducible.
 - (b) Randomly split the sample into a training set and a validation set of equal size.
 - (c) Fit a multiple logistic regression of **default** with predictors **income** and **balance** using only the training observations.
 - (d) Predict the default status of the persons in the validation set using a cutoff value of 0.5.
 - (e) Compute the validation set error which is the fraction of individuals in the test set whose default status is misclassified.
- 3. Repeat the steps in part 2 using set.seed(2) and set.seed(3). Comment on the results.
- 4. Next, consider a logistic regression model that predicts the probability of default using income, balance, and a dummy variable for student. Estimate the test error for this model using the validation set approach with three seeds, set.seed(1), set.seed(2) and set.seed(3). Comment on whether or not including a dummy variable for student leads to a reduction in the test error rate.
- 5. Similar to 4, we consider the same logistic regression model with predictors **income**, **balance**, and a dummy variable for **student**. Now, we use 5-fold cross-validation to estimate the test error rate. Comment on whether or not including a dummy variable for student leads to a reduction in the test error rate.

Problem 2 (16 points)

Solve Problem 9 on page 223 in the textbook "Introduction to Statistical Learning" (second edition).

Problem 3 (14 points)

Solve Problem 1 on page 282 in the textbook "Introduction to Statistical Learning" (second edition).

Problem 4 (4 points)

Consider the following elastic-net problem (which is another extension of lasso)

$$\min_{\beta} \sum_{i=1}^{n} (Y_i - \beta X_i)^2 + \lambda(\alpha \beta^2 + (1 - \alpha)|\beta|),$$

where λ and α are two tuning parameters. For simplicity, we only consider the case that the variable X is univariate, and there is no intercept. Show how one can turn this into a lasso problem using a transformed version of X and Y.