Probabilities and Statistics Refresher

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Axioms of probability - For each event E, we denote P(E) as the probability of event E occuring. By noting E_1, \ldots, E_n mutually exclusive events, we have the 3 following axioms:

- (1) $0 \leq P(E) \leq 1$
- (2) P(S) = 1
- (3) $P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i)$

Permutation - A permutation is an arrangement of r objects from a pool of n objects, in a given order. The number of such arrangements is given by P(n, r), defined as:

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Combination - A combination is an arrangement of r objects from a pool of n objects, where the order does not matter. The number of such arrangements is given by C(n, r), defined as

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}$$

Bayes' rule - For events A and B such that P(B) > 0, we have:

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Partition - Let $\{A_i, i \in \{1, ..., n\}\}$ be such that for all $i, A_i \neq \emptyset$. We say that $\{A_i\}$ is a partition if we have:

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Conditional Probability

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Independence -Two events A and B are independent if and only if we have:

 $P(A \cap B) = P(A)P(B)$

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Cumulative distribution function (CDF) - The cumulative distribution function *F* which is monotonically non-decreasing and is such that $\lim_{x\to-\infty} F(x) = 0$ and $\lim_{x\to+\infty} F(x) = 1$, is defined as:

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Probability density function (PDF) - The probability density function f is the probability that X takes on values between two adjacent realizations of the random variable.

CDF and **PDF** - Properties in the discrete (D) and the continuous (C) cases:

Case	CDF F	PDF f	Properties of PDF
(D)	$F(x) = \sum_{x_i \leqslant x} P(X = x_i)$		$0\leqslant f\left(x_{j} ight)\leqslant 1$ and $\sum_{j}f\left(x_{j} ight)=1$
(C)	$F(x) = \int_{-\infty}^{x} f(y) dy$	$f(x) = \frac{dF}{dx}$	$f(x) \geqslant 0$ and $\int_{-\infty}^{+\infty} f(x) dx = 1$

Variance - The variance of a random variable, often noted Var(X) or σ^2 , is a measure of the spread of its distribution function. It is determined as follows:

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Random Variables

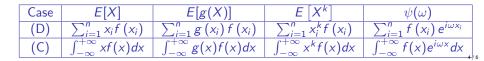
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Expectation and Moments of the Distribution - Here are the expressions of the expected value E[X], generalized expected value E[g(X)], k^{th} moment $E[X^k]$ and characteristic function $\psi(\omega)$ for the discrete and continuous cases:



Conditional density - The conditional density of X w.r.t. Y, noted $f_{X|Y}$, is:

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Covariance - We define the covariance of two random variables X and Y, that we note σ_{XY}^2 or more commonly Cov(X, Y), as follows:

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Correlation - By noting σ_X, σ_Y the standard deviations of X and Y, we define the correlation between the random variables X and Y, noted ρ_{XY} , as follows:

$$\rho_{XY} = \frac{\sigma_{XY}^2}{\sigma_X \sigma_Y}$$

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Parameter estimation

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Sample mean and variance - The sample mean and the sample variance of a random sample are used to estimate the true mean μ and the true variance σ^2 of a distribution, are noted \bar{X} and s^2 respectively, and are such that:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$
 and $s^2 = \hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$.